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Natural Convection & Mixed Convection Heat Transfer Flow of Nanofluid (Ag/TiO_2 – water) in Multi-Shaped Enclosure with Heated Central Circle

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Abstract

Two research investigated the heat transmission of Ag-water and TiO_2-water nanofluids within enclosures of novel geometries, using both natural and mixed convection. The flow is driven by because of temperature differences in natural convection, but in mixed convection, the movement of the wall drives the flow. The NS and energy equations were solved using the finite volume method (FVM), taking into account the Boussinesq approximation. The study investigated how the concentration of nanoparticles and Richardson number affect the flow and heat transfer properties. Unlike previous studies that focused on conventional enclosure shapes, this study explores unique geometries which reveal their role in optimizing heat transport. Nanoparticles often improve heat transport, particularly in settings with moderate to high Richardson numbers. The performance of heat transfer of Ag-water nanofluid is found to be better to that of TiO_2-water. The change from natural convection to mixed convection took place when the Reynolds numbers increased. Mixed convection became the dominant mode at higher values, particularly for larger Grashof numbers. The study's results are consistent with prior studies, providing vital knowledge on how to optimize heat transfer systems that use nanofluids.
Keywords: Natural convection, Mixed Convection, Nanofluid, Heat Transfer, Computational Fluid Dynamics

Introduction

Heat transfer of mixed convection results from the interaction of forced convection and natural convection is of most importance within a range of engineering and geophysical systems. This phenomenon significantly affects energy efficiency in applications such as indoor ventilation, electronic cooling, and the thermal management of reactors and heat exchangers [1]. Buoyancy-driven convection and improving heat transfer in both natural convection and mixed convection settings is necessary for improving energy efficiency. Previous studies have investigated both natural [2] and mixed convection [3] in various applications.

Bilgen et al. [2] examined laminar natural convection within a two-dimensional square cavity where a thin fin connected to the heated wall. They discovered the most effective placement for the fin, usually in the center, results in the minimal transmission of heat by natural convection. Oztop et al. [4] conducted an analysis in square lid-driven cavities and it was found that there is a considerable decrease in heat transmission as the vertical walls move upward in the same direction as opposed to moving in different directions. Khanafer et al. [5] investigated the circumstances of mixed convection heat transport in a hollow with a lid-driven flow. They observed that when the presence of a 2-D circular body in the cavity is positioned close to the bottom wall, it significantly improves heat transmission.

Nanofluids have been proven to greatly improve heat transfer by virtue of their increased thermal conductivity, even at low levels of nanoparticle volume fractions [6]. Esfe et al. [7] and Pal et al. [8] have examined various applications of mixed convection in nanofluids. Specifically, Pal et al. focused on studying on Cu-water nanofluid within a hollow in case of mixed convection. The cavity had a heated sidewall with a thick structure, and an opposite wall that was in motion.

Despite these advancements, the influence of unconventional enclosure geometries on heat transfer, particularly in nanofluid-based systems, remains underexplored. The purpose of the present study is to address this gap by examining the rate of heat transport in natural convection and mixed convection within a differentially heated cavities (DHCs) by integrating the Boussinesq approximation model. The research investigates the impact of Rayleigh number, Richardson number, and Grashof number on heat transmission in multi-shaped enclosures with a center circle, using Ag-water and TiO_2-water nanofluids. To the best of our knowledge, this is the first study to systematically analyze these unique configurations, providing insights for optimizing heat transfer systems.

Problem Statement

The study investigates the characteristics of Ag-water and TiO_2-water nanofluids within two-dimensional cavities of different geometries which is shown in Figure 1. The three geometries under consideration are−Case 1: a two-dimensional domain in the shape of a rhombus with a circular void located at its center. Case 2: a two-dimensional trapezoidal domain, also with a centrally located circular void. Lastly, Case 3: a two-dimensional trapezoidal domain with a concave arc at the top edge, along with a central circular void.The bottom wall and the border of the center void in these cavities are kept at a consistent temperature, while the top wall is thermally insulated. The other two vertical walls are kept at a low temperature. The lower wall is heated to provide a buoyant force. In addition, the upper wall travels in a horizontal direction at a consistent velocity U_0. The flow is assumed to be laminar and fluid properties are constant, except for density variations modeled by the Boussinesq approximation.

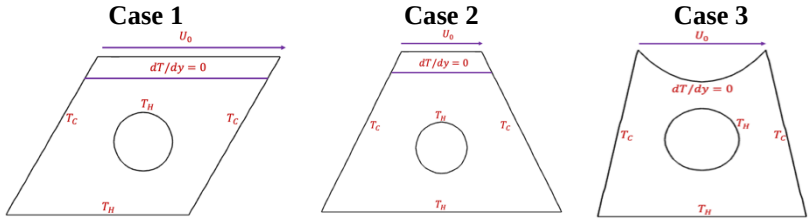


Fig. 1. Physical Geometry

The Navier–Stokes and energy equations determine the incompressible flow which is viscous and temperature distribution within the cavity. The goal here is to explore steady-state solutions by solving the time-independent differential equations. It follows a similar approach to recent research on mixed convection [9]. The thermophysical properties of the base fluid and solid particles are shown in table 1 [10].

Table 1. Properties of base fluid and nanoparticles at 298°K

Property	Water	Ag	TiO_2
C_p (J/kg K)	4179	235	686.2
ρ (kg/m^3)	997.1	10500	4250
κ (W/m K)	0.613	429	8.9538
$\beta^{10^{-5}}$ K	21	1.89	0.9

Mathematical Formulation

The Boussinesq model, which is widely accepted as the standard, is used to estimate fluctuations in fluid density. The Navier-Stokes equations (NSEs) and the energy equation are used to determine the dimensionless numbers that describe the mixed convection issue. The equations for x- and y-momentum, known as the Navier-Stokes equations (NSEs), are explained in detail in reference [11]. These equations are accompanied by the energy equation. The governing equations are:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x} + \nu_{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{1}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y} + \nu_{nf}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial v}{\partial y^2}\right) + \frac{(\rho\beta)_{nf}}{\rho_{nf}}g(T - T_c) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

In this study, T_0 as the temperature of the cold wall, denoted as T_C.

The properties of the hybrid nanofluid are characterized as follows [12]:

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}} \tag{4}$$

$$\rho_{nf} = \varphi\rho_s + (1-\varphi)\rho_f \tag{5}$$

$$\alpha_{nf} = \frac{\kappa_{nf}}{(\rho C_p)_{nf}} \tag{6}$$

$$(\rho C_p)_{nf} = \varphi(\rho C_p)_s + (1-\varphi)(\rho C_p)_f \tag{7}$$

$$(\rho\beta_T)_{nf} = \varphi(\rho\beta_T)_s + (1-\varphi)(\rho\beta_T)_f \tag{8}$$

$$\nu_{nf} = \frac{\mu_{nf}}{\rho_{nf}} \tag{9}$$

The dimensionless parameters are represented as:

$$X = \frac{x}{H}; Y = \frac{y}{H}; U = \frac{u}{U_0}; V = \frac{v}{U_0}; P = \frac{p}{\rho_c U_0}; \theta = \frac{T - T_c}{T_H - T_c} \quad (10)$$

In this context, β_T is the coefficient of thermal expansion, α is the thermal diffusivity, and ρ represents density, all evaluated at the cold temperature T_C (denoted by subscript C). ρ_{nf}, μ_{nf}, κ_{nf} and φ_{nf} represent the density, viscosity, thermal conductivity, and volume fraction of the nanofluid, while ρ_f, μ_f and κ_f denote the corresponding properties of the base fluid.

Using the previously defined nondimensional quantities, the governing equations are:

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\mathrm{Re}}\frac{\nu_{nf}}{\nu_f}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \quad (11)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\mathrm{Re}}\frac{\nu_{nf}}{\nu_c}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ri\theta \quad (12)$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_c}\frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \quad (13)$$

Here, the dimensionless parameters are given as:

$$Gr = \frac{g\beta_c(T_H - T_C)H^3}{\nu_c^2}, Re = \frac{U_0 H}{\nu_c}, Ri = \frac{Gr}{Re^2}, Pr = \frac{\nu_c}{\alpha_c} \quad (14)$$

Now the transformed form of boundary conditions [13]:

$$U(X,1) = 1, U(X,0) = U(0,Y) = U(1,Y) = 0 \quad (15)$$

$$V(X,0) = V(X,1) = V(0,Y) = V(1,Y) = 0 \quad (16)$$

$$\theta(X,0) = 1, \theta(0,Y) = \theta(1,Y) = 0, \frac{\partial \theta}{\partial Y}(X,1) = 0 \quad (17)$$

The boundary of the circle void is heated, $\theta = 1$

The average Nusselt number has been calculated by integrating local Nusselt number and it has been determined along the hot wall of the cavity.

$$\overline{Nu} = \int_0^1 \frac{\kappa_{nf}(\varphi)}{\kappa_f}\frac{\partial \theta}{\partial X} \quad (18)$$

The following Nusselt number ratio is called normalized Nusselt number (Nu*) which is defined as [3]:

$$Nu^* = \frac{\overline{Nu}(\varphi)}{\overline{Nu}(\varphi = 0)} \quad (19)$$

Validations

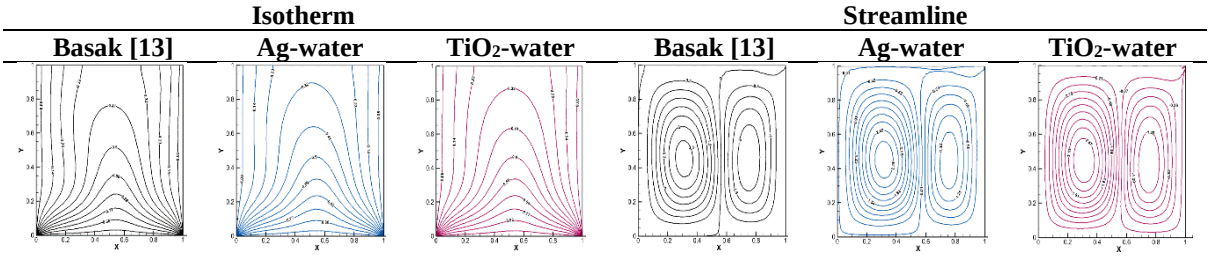


Fig. 2. Validation of the present results for Ag-water and TiO_2-water nanofluids against the data of Basak [13]

The mixed convection scenario with constant characteristics has been validated using reference data from Basak [13]. The outcomes derived from the Finite Volume (FV) technique are represented by black dashed lines. The findings of the current investigation, depicted by blue and red solid lines, are based on simulations utilizing Ag-water and TiO2-water nanofluids. These simulations were conducted using the same characteristics and geometries as those used by Basak [13].

Results and Discussions

The isotherms and streamlines of Ag-water and TiO_2-water have been compared using the physical geometry and boundary conditions provided by Basak. Ag nanoparticles, known for their superior thermal conductivity, exhibit a more pronounced increase in heat transfer rate compared to TiO_2 nanoparticles among those investigated. The impact is more noticeable in Ag-water, as seen in Figure 3.

The study examines the impact of Re, Gr and Ri numbers on the distribution of isotherms and streamlines in these three different cavity geometries filled with Ag-water nanofluid. The flow undergoes a transition from natural convection, characterized by low Reynolds number (Re) and high Richardson number (Ri), to a state of mixed and forced convection, characterized by high Re and low Ri. At low Reynolds number (Re = 1), natural convection

is the dominant mechanism. This is characterized by smooth flow patterns and virtually vertical temperature gradients, indicating a weak heat transfer process driven by buoyancy.

Re=1, Gr=10^4		Re=1, Gr=10^5	
Isotherm	**Streamline**	**Isotherm**	**Streamline**

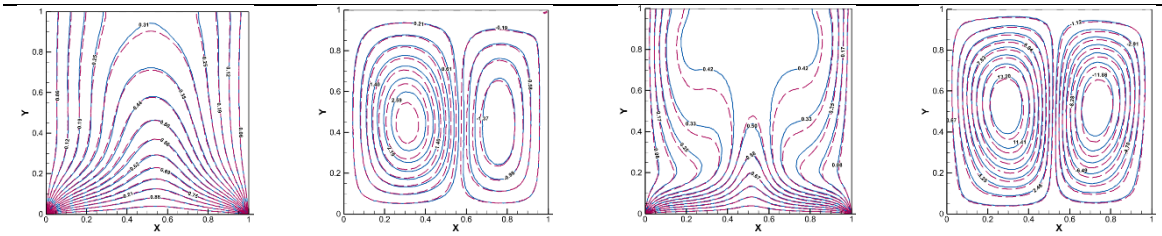
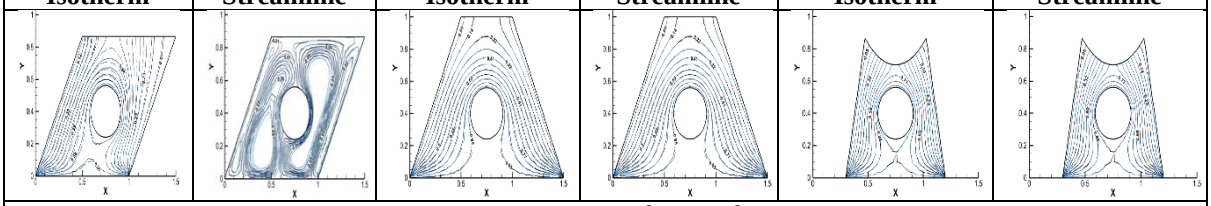
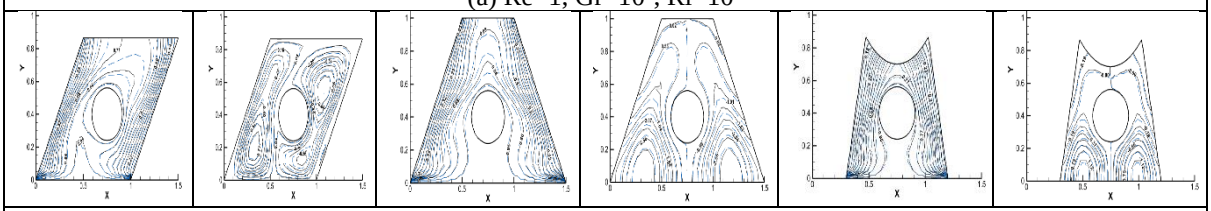
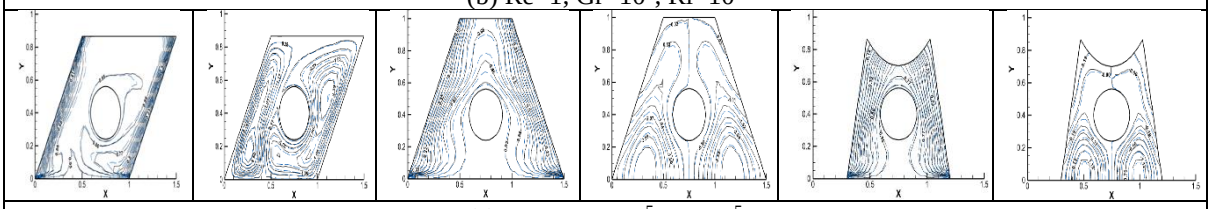
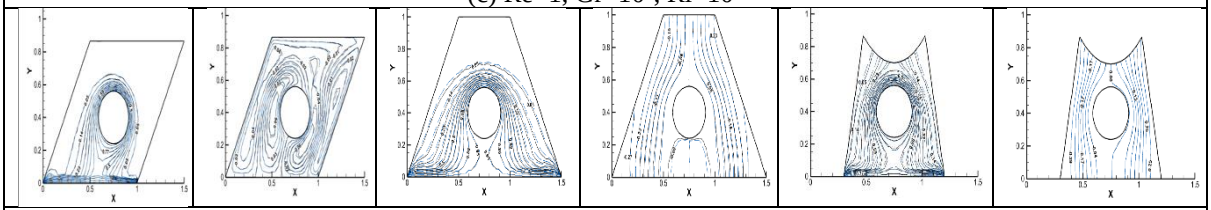
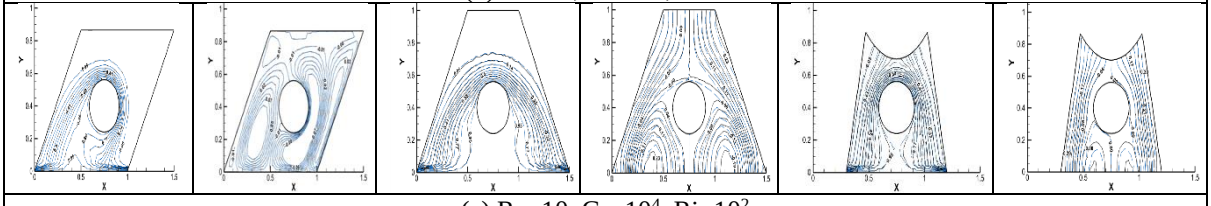


Fig. 3. Comparison of Isotherms and streamlines for Ag-water (solid blue lines) and TiO_2-water (dashed red lines)

The flow pattern becomes more complicated when the Re=10. The lines also represent constant temperature and it shows more curvature which demonstrates the shift towards mixed convection. In Figure 4(d), the flow is considerably influenced by both forced and natural convection. The change is more easily seen in Case 2 as the interaction between natural and forced convection evokes more complex circulation patterns. However, Case 3 depicts diminished buoyancy effects because of its distinct shape.

Case 1		**Case 2**		**Case 3**	
Isotherm	**Streamline**	**Isotherm**	**Streamline**	**Isotherm**	**Streamline**
					
(a) Re=1, Gr=10^3, Ri=10^3					
					
(b) Re=1, Gr=10^4, Ri=10^4					
					
(c) Re=1, Gr=10^5, Ri=10^5					
					
(d) Re=10, Gr=10^3, Ri=10					
					
(e) Re=10, Gr=10^4, Ri=10^2					

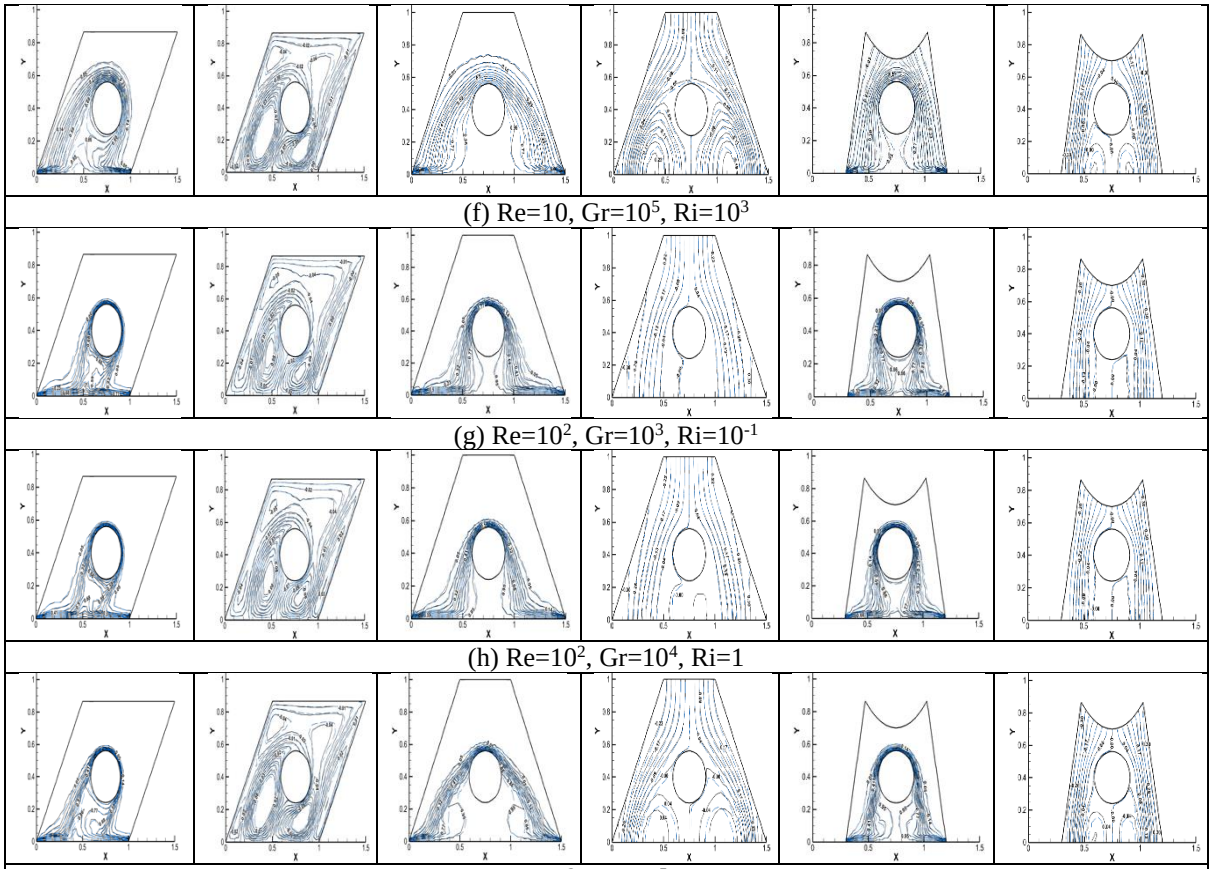


Fig. 4. Isotherms and streamlines for three cavity geometries with 1% (solid) and 8% (dashed) Ag-water nanofluids, showing the effect of mixed convection for Re (1–10^2) and Gr (10^3–10^5)

At Re=10, the dominant forces are inertial forces, resulting in the creation of intense circulation and vortices. The flow is driven by forced convection. The isotherms undergo distortion, hence increasing the efficiency of heat transport. In Case 1, the influence of natural convection is less important compared to the forced one caused by the movement of the lid. However, in Case 3, the concave arc helps to decrease turbulence in the flow, therefore preserving its structure. Ri>10 suggests that natural convection is dominant driven by buoyancy. On the other hand and Ri<0.1 highlights the dominance of forced convection. Nanoparticles have a substantial influence on the transport of heat and the dynamics of fluid flow. The thermal conductivity increases at a low concentration (φ=1%) which causes to smoother temperature distributions and weaker fluid movements.

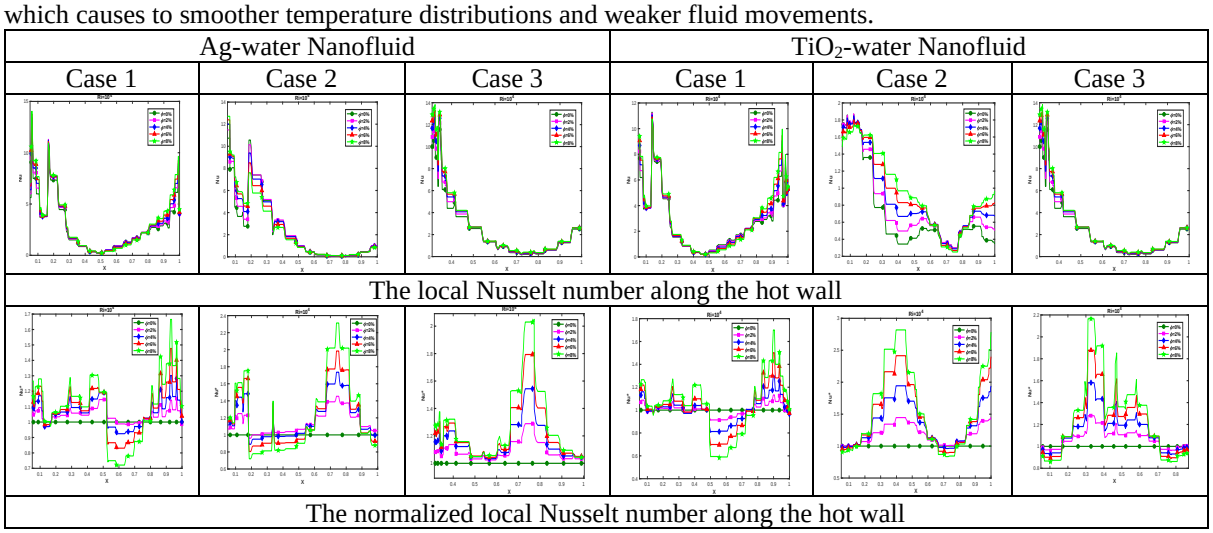


Fig. 5. The local Nusselt number and normalized local Nusselt number along the hot wall for Ag-water and TiO_2-water nanofluid for three cases where $0\% \leq \varphi \leq 8\%$

Slower heat transfer is dominated by natural convection where at a high concentration (φ=8%), the thermal conductivity creates more visible temperature differences which indicates more active heat transfer and dynamic flow patterns.

Figure 5 shows that the local and normalized Nusselt numbers for Ag-water and TiO_2-water nanofluids follow similar patterns throughout the all geometries (Cases 1, 2, and 3). The local Nusselt number consistently falls radially from the central region towards the periphery of the heated wall. In Case 1, for Ag-water, there is a steep decrease in values along the margins but the change is more gradual towards the center. Cases 2 and 3, which involve trapezoidal domains, exhibit greater fluctuations, showing more pronounced variations in heat transmission as a result of changes in cavity form. The heat transfer in TiO_2-water shows a comparable trend, but with consistently lower Nusselt values. Therefore it suggests a less effective heat transfer process. The normalized Nusselt numbers highlight the influence of cavity geometry on nanofluid characteristics. Cases 2 and 3 exhibit higher variations in both fluids, possibly due to intricate flow interactions and increased buoyancy effects. Ag-water has superior heat transfer efficiency across all geometries.

Conclusions

The flow transitions from natural to mixed and forced convection as Re increases. Case 1 demonstrates strong natural convection at lower Re, while Case 3 highlights forced convection at higher Re due to its concave arc. Case 2 balances both and it gradually shifts towards the mixed convection. The results indicate how geometry and flow conditions influence heat transport and flow dynamics in mixed convection phenomenon. Including nanoparticles enhances heat transfer, with silver nanoparticles outperforming titanium oxide. A higher nanoparticle volume fraction improves efficiency by intensifying natural and forced convection, resulting in more vigorous fluid motion within the cavities. Ag-water nanofluid consistently demonstrates higher heat transfer efficiency compared to TiO_2-water across all examined geometries, specially in the rhombus-shaped domain. The cavity shape significantly influences local and normalized Nusselt numbers, with more complex geometries (trapezoidal domains) leading to greater fluctuations and stronger heat transfer variations due to enhanced flow interactions.

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